Advanced Strength and Applied Elasticity (4th Edition)

Step-by-step solution

Step 1 of 14

a)

Determine the principal stresses and related direction cosines for the following case

Write the general expression of the stresses relative to an *x, y, z* coordinate system be

$$\begin{bmatrix}\sigma_x & \tau_{xy} & \tau_{xz}\\ \tau_{xy} & \sigma_y & \tau_{yz}\\ \tau_{xz} & \tau_{yz} & \sigma_z\end{bmatrix}=\begin{bmatrix}3 & 4 & 6\\ 4 & 2 & 5\\ 6 & 5 & 1\end{bmatrix}\text{MPa}$$

Here,

Stresses acting in the *x, y, z* coordinate system, σ_x, σ_y and σ_z

Shear stresses acting in the *xy*- plane, τ_{xy}

Shear stresses acting in the *xz*- plane, τ_{xz}

Shear stresses acting in the *yz*- plane, τ_{yz}

Comment

Step 2 of 14

Determine the stress invariants using equation 1.34.

Find stress invariant I_1.

$$I_1=\sigma_x+\sigma_y+\sigma_z$$

Substitute $3\,\text{MPa}$ for σ_x, $2\,\text{MPa}$ for σ_y, $1\,\text{MPa}$ for σ_z

$$I_1=3+2+1$$
$$=6\,\text{MPa}$$

Find stress invariant I_2.

$$I_2=\sigma_x\sigma_y+\sigma_x\sigma_z+\sigma_y\sigma_z-\tau_{xy}^2-\tau_{xz}^2-\tau_{yz}^2$$

Substitute $3\,\text{MPa}$ for σ_x, $2\,\text{MPa}$ for σ_y, $1\,\text{MPa}$ for σ_z, $4\,\text{MPa}$ for τ_{xy}, $6\,\text{MPa}$ for τ_{yz}, $5\,\text{MPa}$ for τ_{xz}

$$I_2=(3)(2)+(3)(1)+(2)(1)-4^2-6^2-5^2$$
$$=-66\,\text{MPa}$$

Find stress invariant I_3.

$$I_3=\begin{vmatrix}\sigma_x & \tau_{xy} & \tau_{xz}\\ \tau_{xy} & \sigma_y & \tau_{yz}\\ \tau_{xz} & \tau_{yz} & \sigma_z\end{vmatrix}$$

Substitute $3\,\text{MPa}$ for σ_x, $2\,\text{MPa}$ for σ_y, $1\,\text{MPa}$ for σ_z, $4\,\text{MPa}$ for τ_{xy}, $6\,\text{MPa}$ for τ_{yz}, $5\,\text{MPa}$ for τ_{xz}

$$I_3=\begin{vmatrix}3 & 4 & 6\\ 4 & 2 & 5\\ 6 & 5 & 1\end{vmatrix}$$
$$=83\,\text{MPa}$$

Comment

Step 3 of 14

Determine the principal stresses $\boldsymbol{\sigma_p}$ using equation 1.33.

$$\boldsymbol{\sigma_p^3-I_1\sigma_p^2+I_2\sigma_p-I_3=0}$$

Substitute 6 for I_1, -66 for I_2 and 83 for I_3

$$\sigma_p^3-6\sigma_p^2-66\sigma_p-83=0$$

Solving the above equation we get three values for σ_p,

$$\sigma_p=-4.53\,\text{MPa}$$
$$\sigma_p=-1.52\,\text{MPa}$$
$$\sigma_p=12.05\,\text{MPa}$$

There are 3 solutions or 3 principal stresses.

Let the principal stresses be denoted by $\boldsymbol{\sigma_1}$, $\boldsymbol{\sigma_2}$, and $\boldsymbol{\sigma_3}$ where $\boldsymbol{\sigma_1>\sigma_2>\sigma_3}$.

Therefore the principal stress σ_1 is $\boxed{12.05\text{ MPa}}$, σ_2 is $\boxed{-1.52\text{ MPa}}$, and $\boldsymbol{\sigma_3}$ is $\boxed{-4.53\text{ MPa}}$

Comment

Step 4 of 14

Determine the related direction cosines for the principal stress σ_1 = 12.05 MPa using the method of Direction Cosines.

Determine the cofactor determinant a_1.

$$a_1=\begin{vmatrix}(\sigma_y-\sigma_1) & \tau_{yz}\\ \tau_{yz} & (\sigma_z-\sigma_1)\end{vmatrix}$$

Substitute 12.05 MPa for σ_1, $2\,\text{MPa}$ for σ_y, $1\,\text{MPa}$ for σ_z, $4\,\text{MPa}$ for τ_{xy}, $6\,\text{MPa}$ for τ_{yz}, $5\,\text{MPa}$ for τ_{xz}.

$$a_1=\begin{vmatrix}(2-12.05) & 5\\ 5 & (1-12.05)\end{vmatrix}$$
$$=86.05\,\text{MPa}$$

Determine the cofactor determinant b_1.

$$b_1=-\begin{vmatrix}\tau_{xy} & \tau_{yz}\\ \tau_{xz} & (\sigma_z-\sigma_1)\end{vmatrix}$$

Substitute 12.05 MPa for σ_1, $2\,\text{MPa}$ for σ_y, $1\,\text{MPa}$ for σ_z, $4\,\text{MPa}$ for τ_{xy}, $6\,\text{MPa}$ for τ_{yz}, $5\,\text{MPa}$ for τ_{xz}.

$$b_1=-\begin{vmatrix}4 & 5\\ 6 & 1-12.05\end{vmatrix}$$
$$=74.2\,\text{MPa}$$

Determine the cofactor determinant c_1.

$$c_1=\begin{vmatrix}\tau_{xy} & (\sigma_y-\sigma_1)\\ \tau_{xz} & \tau_{yz}\end{vmatrix}$$

Substitute 12.05 MPa for σ_1, $2\,\text{MPa}$ for σ_y, $1\,\text{MPa}$ for σ_z, $4\,\text{MPa}$ for τ_{xy}, $6\,\text{MPa}$ for τ_{yz}, $5\,\text{MPa}$ for τ_{xz}.

$$c_1=\begin{vmatrix}4 & 2-12.05\\ 6 & 5\end{vmatrix}$$
$$=80.3\,\text{MPa}$$

Comment

Step 5 of 14

Define the constant k_1

$$k_1=\frac{1}{\sqrt{a_1^2+b_1^2+c_1^2}}$$

Substitute $80.3\,\text{MPa}$ for c_1, $74.2\,\text{MPa}$ for b_1 and $86.05\,\text{MPa}$ for a_1

$$k_1=\frac{1}{\sqrt{86.05^2+74.2^2+80.3^2}}$$
$$=0.007187$$

Determine the direction cosines l_1.

$$l_1=a_1k_1$$

Substitute 0.007187 for k_1, $86.05\,\text{MPa}$ for a_1

$$l_1=(86.05)(0.007187)$$
$$=0.618$$

Determine the direction cosines m_1.

$$m_1=b_1k_1$$

Substitute 0.007187 for k_1, $74.2\,\text{MPa}$ for b_1

$$m_1=(74.2)(0.007187)$$
$$=0.533$$

Determine the direction cosines n_1.

$$n_1=c_1k_1$$

Substitute 0.007187 for k_1, $80.3\,\text{MPa}$ for c_1

$$n_1=(80.3)(0.007187)$$
$$=0.577$$

Verify the solution for the direction cosines.

$$\boldsymbol{l_1^2+m_1^2+n_1^2=1}$$
$$\boldsymbol{0.618^2+(-0.533)^2+0.577^2=1}$$
$$\boldsymbol{1=1}$$

Therefore the direction of the principal stress s_1 is $\boxed{\begin{bmatrix}0.618\\ 0.533\\ 0.577\end{bmatrix}}$

Comment

Step 6 of 14

Determine the direction cosines for the principal stress $\sigma_2=-1.52\,\text{MPa}$

Determine the cofactor determinant a_2.

$$a_2=\begin{vmatrix}(\sigma_y-\sigma_2) & \tau_{yz}\\ \tau_{yz} & (\sigma_z-\sigma_2)\end{vmatrix}$$

Substitute -1.52 MPa for σ_2, $2\,\text{MPa}$ for σ_y, $1\,\text{MPa}$ for σ_z, $4\,\text{MPa}$ for τ_{xy}, $6\,\text{MPa}$ for τ_{yz}, $5\,\text{MPa}$ for τ_{xz}.

$$a_2=\begin{vmatrix}2+1.52 & 5\\ 5 & 1+1.52\end{vmatrix}$$
$$=-16.13$$

Determine the cofactor determinant b_2.

$$b_2=-\begin{vmatrix}\tau_{xy} & \tau_{yz}\\ \tau_{xz} & (\sigma_z-\sigma_2)\end{vmatrix}$$

Substitute -1.52 MPa for σ_2, $2\,\text{MPa}$ for σ_y, $1\,\text{MPa}$ for σ_z, $4\,\text{MPa}$ for τ_{xy}, $6\,\text{MPa}$ for τ_{yz}, $5\,\text{MPa}$ for τ_{xz}.

$$b_2=-\begin{vmatrix}4 & 5\\ 6 & 1+1.52\end{vmatrix}$$
$$=19.92$$

Determine the cofactor determinant c_2.

$$c_2=\begin{vmatrix}\tau_{xy} & (\sigma_y-\sigma_2)\\ \tau_{xz} & \tau_{yz}\end{vmatrix}$$

Substitute -1.52 MPa for σ_2, $2\,\text{MPa}$ for σ_y, $1\,\text{MPa}$ for σ_z, $4\,\text{MPa}$ for τ_{xy}, $6\,\text{MPa}$ for τ_{yz}, $5\,\text{MPa}$ for τ_{xz}.

$$c_2=\begin{vmatrix}4 & 2+1.52\\ 6 & 5\end{vmatrix}$$
$$=-1.12$$

Define the constant k_2

$$k_2=\frac{1}{\sqrt{a_2^2+b_2^2+c_2^2}}$$

Substitute -1.12 for c_2, 19.92 for b_2, -16.13 for a_2

$$k_2=\frac{1}{\sqrt{(-16.13)^2+19.92^2+(-1.12)^2}}$$
$$=0.038977$$

Comment

Step 7 of 14

Determine the direction cosines l_2.

$$l_2=a_2k_2$$

Substitute 0.038977 for k_2, -16.13 for a_2

$$l_2=(-16.13)(0.038977)$$
$$=-0.629$$

Determine the direction cosines m_2.

$$m_2=b_2k_2$$

Substitute 0.038977 for k_2, 19.92 for b_2

$$m_2=(19.92)(0.038977)$$
$$=0.776$$

Determine the direction cosines n_2.

$$n_2=c_2k_2$$

Substitute 0.038977 for k_2, -1.12 for c_2

$$n_2=(-1.12)(0.038977)$$
$$=-0.044$$

Verify the solution for the direction cosines.

$$\boldsymbol{l_2^2+m_2^2+n_2^2=1}$$
$$\boldsymbol{(-0.629)^2+0.776^2+(-0.044)^2=1}$$
$$\boldsymbol{1=1}$$

Therefore the direction of the principal stress s_2 is $\boxed{\begin{bmatrix}-0.629\\ 0.776\\ -0.044\end{bmatrix}}$

Comment

Step 8 of 14

Now determine the related direction cosines for the principal stress $\sigma_3=-4.53\text{ MPa}$

Determine the cofactor determinant a_3.

$$a_3=\begin{vmatrix}(\sigma_y-\sigma_3) & \tau_{yz}\\ \tau_{yz} & (\sigma_z-\sigma_3)\end{vmatrix}$$

Substitute -4.53 MPa for σ_3, $2\,\text{MPa}$ for σ_y, $1\,\text{MPa}$ for σ_z, $4\,\text{MPa}$ for τ_{xy}, $6\,\text{MPa}$ for τ_{yz}, $5\,\text{MPa}$ for τ_{xz}.

$$a_3=\begin{vmatrix}2+4.53 & 5\\ 5 & 1+4.53\end{vmatrix}$$
$$=11.11$$

Determine the cofactor determinant b_3.

$$b_3=-\begin{vmatrix}\tau_{xy} & \tau_{yz}\\ \tau_{xz} & (\sigma_z-\sigma_3)\end{vmatrix}$$

Substitute -4.53 MPa for σ_3, $2\,\text{MPa}$ for σ_y, $1\,\text{MPa}$ for σ_z, $4\,\text{MPa}$ for τ_{xy}, $6\,\text{MPa}$ for τ_{yz}, $5\,\text{MPa}$ for τ_{xz}.

$$b_3=-\begin{vmatrix}4 & 5\\ 6 & 1+4.53\end{vmatrix}$$
$$=7.88$$

Determine the cofactor determinant c_3.

$$c_3=\begin{vmatrix}\tau_{xy} & (\sigma_y-\sigma_1)\\ \tau_{xz} & \tau_{yz}\end{vmatrix}$$

Substitute -4.53 MPa for σ_3, $2\,\text{MPa}$ for σ_y, $1\,\text{MPa}$ for σ_z, $4\,\text{MPa}$ for τ_{xy}, $6\,\text{MPa}$ for τ_{yz}, $5\,\text{MPa}$ for τ_{xz}.

$$c_3=\begin{vmatrix}4 & 2+4.53\\ 6 & 5\end{vmatrix}$$
$$=-19.18$$

Define the constant k_3

$$k_3=\frac{1}{\sqrt{a_3^2+b_3^2+c_3^2}}$$

Substitute -19.18 for c_3, 7.88 for b_3 and 11.11 for a_3

$$k_3=\frac{1}{\sqrt{11.11^2+7.88^2+(-19.18)^2}}$$
$$=0.042509$$

Determine the direction cosines l_3.

$$l_3=a_3k_3$$

Substitute 0.042509 for k_3, 11.11 for a_3

$$l_3=(11.11)(0.042509)$$
$$=0.472$$

Determine the direction cosines m_3.

$m_3=b_3k_3$ Substitute 0.042509 for k_3, 7.88 for b_3

$$m_3=(7.88)(0.042509)$$
$$=0.335$$

Determine the direction cosines n_3.

$$n_3=c_3k_3$$

Substitute 0.042509 for k_3, -19.18 for c_3

$$n_3=(-19.18)(0.042509)$$
$$=-0.815$$

Verify the solution for the direction cosines.

$$\boldsymbol{l_3^2+m_3^2+n_3^2=1}$$
$$\boldsymbol{0.472^2+0.335^2+(-0.815)^2=1}$$
$$\boldsymbol{1=1}$$

Therefore the direction of the principal stress s_3 is $\boxed{\begin{bmatrix}0.472\\ 0.335\\ -0.815\end{bmatrix}}$

Comment

Step 9 of 14

b)

Obtain principal stresses and related direction cosines for the following case

$$\begin{bmatrix}\sigma_x & \tau_{xy} & \tau_{xz}\\ \tau_{xy} & \sigma_y & \tau_{yz}\\ \tau_{xz} & \tau_{yz} & \sigma_z\end{bmatrix}=\begin{bmatrix}14.32 & 0.8 & 1.55\\ 0.8 & 6.97 & 5.2\\ 1.55 & 5.2 & 16.3\end{bmatrix}\textbf{MPa}$$

Comment

Step 10 of 14

Determine the stress invariants using equation 1.34.

Find stress invariant I_1.

$$\boldsymbol{I_1=\sigma_x+\sigma_y+\sigma_z}$$
$$=\mathbf{14.32+6.97+16.3}$$
$$=\mathbf{37.59}$$

Find stress invariant I_2.

$$\boldsymbol{I_2=\sigma_x\sigma_y+\sigma_x\sigma_z+\sigma_y\sigma_z-\tau_{xy}^2-\tau_{xz}^2-\tau_{yz}^2}$$
$$=\mathbf{(14.32)(6.97)+(14.32)(16.3)+(6.97)(16.3)-0.8^2-1.55^2-5.2^2}$$
$$=\mathbf{416.755}$$

Find stress invariant I_3.

$$\boldsymbol{I_3=\begin{vmatrix}\sigma_x & \tau_{xy} & \tau_{xz}\\ \tau_{xy} & \sigma_y & \tau_{yz}\\ \tau_{xz} & \tau_{yz} & \sigma_z\end{vmatrix}}$$
$$=\begin{vmatrix}14.32 & 0.8 & 1.55\\ 0.8 & 6.97 & 5.2\\ 1.55 & 5.2 & 16.3\end{vmatrix}$$
$$=\mathbf{1225.42}$$

Comment

Step 11 of 14

Determine the principal stresses $\boldsymbol{\sigma_p}$ using equation 1.33.

$$\boldsymbol{\sigma_p^3-I_1\sigma_p^2+I_2\sigma_p-I_3=0}$$

Substitute 37.59 for I_1, 416.755 for I_2 and 1225.42 for I_3.

$$\boldsymbol{\sigma_p^3-37.95\sigma_p^2+416.755\sigma_p-1225.42=0}$$
$$\boldsymbol{\sigma_p=20.62}$$
$$\boldsymbol{\sigma_p=12.62}$$
$$\boldsymbol{\sigma_p=4.71}$$

Notice there are 3 solutions or 3 principal stresses.

Let the principal stresses be denoted by $\boldsymbol{\sigma_1}$, $\boldsymbol{\sigma_2}$, and $\boldsymbol{\sigma_3}$ where $\boldsymbol{\sigma_1>\sigma_2>\sigma_3}$.

Therefore the principal stress s_1 is $\boxed{20.62\text{ MPa}}$, s_2 is $\boxed{12.62\text{ MPa}}$, and s_3 is $\boxed{4.71\text{ MPa}}$

Comment

Step 12 of 14

Now determine the related direction cosines for the principal stress s_1 = 20.62 MPa using the method outlined in Appendix B.2 Direction Cosines.

Determine the cofactor a_1.

$$\boldsymbol{a_1=\begin{vmatrix}(\sigma_y-\sigma_1) & \tau_{yz}\\ \tau_{yz} & (\sigma_z-\sigma_1)\end{vmatrix}}$$
$$=\begin{vmatrix}6.97-20.62 & 5.2\\ 5.2 & 16.3-20.62\end{vmatrix}$$
$$=\mathbf{31.93}$$

Determine the cofactor b_1.

$$\boldsymbol{b_1=-\begin{vmatrix}\tau_{xy} & \tau_{yz}\\ \tau_{xz} & (\sigma_z-\sigma_1)\end{vmatrix}}$$
$$=-\begin{vmatrix}0.8 & 5.2\\ 1.55 & 16.3-20.62\end{vmatrix}$$
$$=\mathbf{11.52}$$

Determine the cofactor c_1.

$= 25.32$

Define the constant k_1

$$k_1 = \frac{1}{\sqrt{a_1^2 + b_1^2 + c_1^2}}$$
$$= \frac{1}{\sqrt{31.93^2 + 11.52^2 + 25.32^2}}$$
$$= 0.023614$$

Determine the direction cosines l_1.

$$l_1 = a_1 k_1$$
$$= (31.93)(0.023614)$$
$$= 0.754$$

Determine the direction cosines m_1.

$$m_1 = b_1 k_1$$
$$= (11.52)(0.023614)$$
$$= 0.272$$

Determine the direction cosines n_1.

$$n_1 = c_1 k_1$$
$$= (80.3)(0.023614)$$
$$= 0.598$$

Verify the solution for the direction cosines.

$$l_1^2 + m_1^2 + n_1^2 = 1$$
$$0.754^2 + 0.272^2 + 0.598^2 = 1$$
$$1 = 1$$

Therefore the direction of the principal stress s_1 is $\boxed{\begin{bmatrix} 0.754 \\ 0.272 \\ 0.598 \end{bmatrix}}$

Comment

Step 13 of 14

Now determine the related direction cosines for the principal stress s_2 = 12.62 MPa using the method outlined in Appendix B.2 Direction Cosines.

Determine the cofactor a_2.

$$a_2 = \begin{vmatrix} (\sigma_y - \sigma_2) & \tau_{yz} \\ \tau_{yz} & (\sigma_z - \sigma_2) \end{vmatrix}$$
$$= \begin{vmatrix} 6.97 - 12.62 & 5.2 \\ 5.2 & 16.3 - 12.62 \end{vmatrix}$$
$$= -47.83$$

Determine the cofactor b_2.

$$b_2 = -\begin{vmatrix} \tau_{xy} & \tau_{yz} \\ \tau_{xz} & (\sigma_z - \sigma_2) \end{vmatrix}$$
$$= -\begin{vmatrix} 0.8 & 5.2 \\ 1.55 & 16.3 - 12.62 \end{vmatrix}$$
$$= 5.12$$

Determine the cofactor c_2.

$$c_2 = \begin{vmatrix} \tau_{xy} & (\sigma_y - \sigma_2) \\ \tau_{xz} & \tau_{yz} \end{vmatrix}$$
$$= \begin{vmatrix} 0.8 & 6.97 - 12.62 \\ 1.55 & 5.2 \end{vmatrix}$$
$$= 12.92$$

Define the constant k_2

$$k_2 = \frac{1}{\sqrt{a_2^2 + b_2^2 + c_2^2}}$$
$$= \frac{1}{\sqrt{(-47.83)^2 + 5.12^2 + 12.92^2}}$$
$$= 0.020077$$

Determine the direction cosines l_2.

$$l_2 = a_2 k_2$$
$$= (-47.83)(0.020077)$$
$$= -0.960$$

Determine the direction cosines m_2.

$$m_2 = b_2 k_2$$
$$= (5.12)(0.020077)$$
$$= 0.103$$

Determine the direction cosines n_2.

$$n_2 = c_2 k_2$$
$$= (12.92)(0.020077)$$
$$= 0.259$$

Verify the solution for the direction cosines.

$$l_2^2 + m_2^2 + n_2^2 = 1$$
$$(-0.960)^2 + 0.103^2 + 0.259^2 = 1$$
$$1 = 1$$

Therefore the direction of the principal stress s_2 is $\boxed{\begin{bmatrix} -0.960 \\ 0.103 \\ 0.259 \end{bmatrix}}$

Comment

Step 14 of 14

Now determine the related direction cosines for the principal stress s_3 = 4.71 MPa using the method outlined in Appendix B.2 Direction Cosines.

Determine the cofactor a_3.

$$a_3 = \begin{vmatrix} (\sigma_y - \sigma_3) & \tau_{yz} \\ \tau_{yz} & (\sigma_z - \sigma_3) \end{vmatrix}$$
$$= \begin{vmatrix} 6.97 - 4.71 & 5.2 \\ 5.2 & 16.3 - 4.71 \end{vmatrix}$$
$$= -0.85$$

Determine the cofactor b_3.

$$b_3 = -\begin{vmatrix} \tau_{xy} & \tau_{yz} \\ \tau_{xz} & (\sigma_z - \sigma_3) \end{vmatrix}$$
$$= -\begin{vmatrix} 0.8 & 5.2 \\ 1.55 & 16.3 - 4.71 \end{vmatrix}$$
$$= -1.21$$

Determine the cofactor c_3.

$$c_2 = \begin{vmatrix} \tau_{xy} & (\sigma_y - \sigma_3) \\ \tau_{xz} & \tau_{yz} \end{vmatrix}$$
$$= \begin{vmatrix} 0.8 & 6.97 - 4.71 \\ 1.55 & 5.2 \end{vmatrix}$$
$$= 0.66$$

Define the constant k_3

$$k_3 = \frac{1}{\sqrt{a_3^2 + b_3^2 + c_3^2}}$$
$$= \frac{1}{\sqrt{(-0.85)^2 + (-1.21)^2 + 0.66^2}}$$
$$= 0.617543$$

Determine the direction cosines l_3.

$$l_3 = a_3 k_3$$
$$= (-0.85)(0.617543)$$
$$= -0.525$$

Determine the direction cosines m_3.

$$m_3 = b_3 k_3$$
$$= (-1.21)(0.617543)$$
$$= 0.747$$

Determine the direction cosines n_3.

$$n_3 = c_3 k_3$$
$$= (0.66)(0.617543)$$
$$= 0.408$$

Verify the solution for the direction cosines.

$$l_3^2 + m_3^2 + n_3^2 = 1$$
$$(-0.525)^2 + (-0.747)^2 + 0.408^2 = 1$$
$$1 = 1$$

Therefore the direction of the principal stress s_3 is $\boxed{\begin{bmatrix} -0.525 \\ -0.747 \\ 0.408 \end{bmatrix}}$

Comment

Was this solution helpful?

Recommended solutions for you in Chapter 1